Hyperspectral Sub-Pixel Target Identification using Least-Angle Regression

Pierre V. Villeneuve, Alex R. Boisvert and Alan D. Stocker
Space Computer Corporation, 12121 Wilshire Blvd, Suite 910, Los Angeles, CA

ABSTRACT
A novel approach to VNIR hyperspectral target identification is presented based on the Least-Angle Regression (LARS) variable selection and model building algorithm. The problem to be solved is that of accurately identifying a target’s primary signature component given a sub-pixel observation. Traditional matched detectors (MF, ACE, etc.) perform well at discriminating a target from a random cluttered background, but do not perform so well at unambiguously matching an observation with its counterpart in a large spectral library containing thousands of signatures. The LARS model-building algorithm efficiently selects a parsimonious subset of a large ensemble of model terms to optimally describe a particular target observation. The LARS solution technique is a recent addition to the family of model selection algorithms that includes Stepwise Regression, Forward Selection, and Backward Elimination. LARS is particularly well-suited to this problem as it is easily modified to enforce material abundance constraints: positive coefficients that sum to unity. Other approaches generally enforce such constraints in an ad-hoc fashion or use computationally demanding nonlinear programming solution techniques. LARS enforces these constraints as an inherent property of the model while remaining as computationally efficient as traditional sequential linear least-squares solvers. We demonstrate and quantify sub-pixel material identification performance using simulated target observations tested against large signature libraries.

1. INTRODUCTION
Airborne VNIR-SWIR hyperspectral sensors are an enabling technology for tactical target detection scenarios. On-board real-time automated data processing systems generally implement one or more variants of anomaly (RX) or matched signature (MF, ACE) detection algorithms (Manolakis, 2002; Reed, 1974). These algorithms are easily shown to be optimal in situations where Gaussian noise and clutter interference is a valid assumption. Unfortunately, real scenes observed with real sensors often show distributions with noticeable “fat” tails (Theiler, 2009). This in turn results in larger-than-expected false alarm rates for a practical target detection rate. Current deployed systems use a co-registered high-resolution panchromatic imager to provide contextual information an operator may use to decide whether or not a cued detection result deserves further investigation (Stevenson, 2005). The combination of real-time continuous detection processing, rapid scan rates and large search areas often results in more target cues than an operator can handle in a reasonable amount of time (Eismann, 2010).

Examples of factors that affect overall target detection performance include: 1) how well a library signature is matched to the in-scene observable; 2) how well background clutter can be estimated, and 3) how well the signature and clutter information are used by an algorithm to maximize the target’s signal-to-clutter ratio. There is no guarantee that a technique that is optimal for detection is also optimal for identification. The important factors for identification relate to how well an observed target spectrum can be unambiguously matched to its counterpart in a large spectral library containing thousands of material signatures. A simple-minded view of the difference between detection and identification is as follows: detection performance is driven by background clutter and its projection onto a target filter; while identification performance is driven by the complexity of the library (e.g. number and similarity of member signatures) and how accurately an algorithm can match the observation to any of the library signatures. Note that an “accurately” model is not inherently a good or best-fitting model, if in fact the model’s signature is not a proper match to the observed signal. An acceptable algorithm should not “over-fit” the model to the observable and should restrict the solution to physically-meaningful parameters (e.g. do not allow negative amounts of materials, etc.).

This report describes the GeoID algorithm for sub-pixel target identification using a constrained linear mixture model combined with a multiple-hypothesis testing framework. Material identification is an application unto itself, in that it is desirable to apply a label to a detected target cue. A more advanced application is to feed the ID results back to the
detection system and alternatively reject or accept a target based on the outcome of the identification process. A prime example of this includes false-alarm mitigation (FAM) where the objective is to reduce the total number of false alarms without sacrificing target detection sensitivity. The system operator is then only presented with detection cues that have passed a two stage process of detection plus identification.

**Target Identification**
- Observe target mixed with background
- Characterize background with in-scene data
- Fit observation with each signature in turn plus background spectra
- Rank library signatures by goodness-of-fit

**Rank single-target models by fitting error**
1st + + + Good Fit
2nd ++ + Poor Fit
3rd ++ + Poor Fit

Figure 1. Target identification concept where a large library of material signatures are tested one at a time in a model mixed with scene-based background components.

Figure 1 is an overview of the GeoID identification concept applied to a detected target cue. The observed signal will likely be a mixture of the target’s inherent signature plus a weighted combination of other background signatures, such as soil, vegetation, or other man-made materials. The non-target background scene area is characterized in terms of first and second-order statistics plus data-derived endmember basis functions. A linear mixture model is constructed for each signature in a relatively large library in combination with all available background endmembers. The full set of library signatures are ranked in order of “goodness-of-fit”. If the material signature used to generate the original matched detection results is not present in the top-ranked ID results, then the observed target is considered a likely false alarm and is rejected.

2. GEOID TARGET MODEL

2.1. Linear Mixture Model
A target-plus-background mixture model is constructed as a linear combination of multiple background basis functions and a single library material signature. It is assumed that the hyperspectral data has been first corrected from radiance to reflectance and that the signature library is also in units of reflectance. Examples of software tools and algorithms for converting radiance to reflectance include FLAASH (Adler-Golden, 1999), QUAC (Bernstein, 2005), and Empirical Line Method. The model coefficients represent the material fill fraction in the observed target signal. A physically realistic solution to this mixture model is one where the coefficients are positive and sum to a number less than or equal to unity. Allowing for a sum less than 100% is needed to account for variable illumination effects in the scene not
removed during a radiance-to-reflectance correction procedure. Figure 2 illustrates how a constrained linear mixture model more appropriately describes a sub-pixel target than an unconstrained model. The observed signal is a mixture of the target signature (top right) and the two background components (top left and bottom right). The constrained fit of the null-hypothesis model (left panel) shows the best fit of the simplex (a line in this case) to the target observation. The residual fitting error is indicated by the dashed line between the model fit and the observation. The center panel illustrates a target model constructed with an incorrect material signature. The unconstrained solution in this two-dimensional representation would yield a perfect fit to the target observation, however, this would require negative amount of this incorrect library material. A constrained fit yields a solution point on the edge of the model simplex (now a triangle) that is closest to the target observation. Finally, the right panel illustrates the model fit using the correct material signature. The result is a perfect fit in this two-dimensional representation; however extension to many more dimensions (e.g. 100 or more for hyperspectral data) will not be so clear-cut.

A model constructed from a random incorrect material signature will possibly show a residual error that is at some level better than the background-only null-hypothesis model. A material library containing thousands of signatures is guaranteed to have a certain degree of correlation amongst its constituents. Thus a model constructed with an incorrect signature can still potentially yield a significantly improved fit relative to the null hypothesis. However, a model based on the correct signature will generate a better fitting model, and will rank higher using a metric based on the models’ goodness of fit relative to the null-hypothesis scenario.

2.2. LARS / LASSO as a Constrained Model Solver

Sub-pixel target exploitation requires physically meaningful estimates for material fill fractions, for both background components and library signatures. This requires that model coefficients have positive values that sum to less than or equal to unity. Common available solutions include:

1) **Ignore the problem**: use an unconstrained linear solver.

2) **Post-process**: use an unconstrained solver, but enforce constraints via post-processing normalization.

3) **Partially-constrained**: use sum-constrained least-squares with sum of the coefficients forced to unity.

4) **Partially-constrained**: use non-negative least-squares (NNLS) method to force positive coefficients.

5) **Fully-constrained**: simultaneously enforce sum constraint and sign constraint.

Figure 2. Constrained linear mixture model used to compare many signature + background models to a background-only model. Linear fit is constrained to the multi-dimensional simplex spanning the model basis functions such that coefficients are positive and whose sum is less than or equal to unity.
The recently developed Least-Angle Regression (LARS) algorithm (Efron, 2003) with the LASSO extension (Tibshirani, 1996) provides an efficient solution to minimizing the fitting error, subject to an $l_1$-norm constrain:

$$\hat{\beta} = \arg \min_{\beta} \| y - X\beta \|^2 : \sum_{j=1}^{p} |\beta_j| \leq \gamma,$$

where $y$ is the observed target spectrum, $X$ is a matrix whose column vectors are the model basis functions, and $\hat{\beta}$ is the vector of model coefficients corresponding to the “abundance” of each model term in the observed target signal. The parameter $\gamma$ is the constraint on the sum of coefficient absolute values.

The LARS algorithm (Figure 3) is an improvement over traditional stepwise forwards / backwards linear model solvers. Basis functions are added to an “active set” such that they all observe the same angle relative to the residual error vector. New terms are only added to the model when they are able to reduce fitting error at least as well as those in the active set. The model is thus constructed progressively and parsimoniously as the solution follows a democratic path while minimizing fitting error. Much attention in the literature has been focused on the use of LARS as a method for efficiently deriving sparse models.

The LASSO extension to LARS is based on a more general-purpose technique for solving linear models subject to a $l_1$-norm constraint, whereby the absolute values of a model’s coefficients are required to sum to less than or equal to a certain value. The implementation of LASSO within the LARS framework is remarkably simple: check for a change in a coefficient’s sign. When such a change is detected, the solution path is reversed to the point where the coefficient is equal to zero, and the corresponding model term is dropped from the active set. The model solution continues forward...
from this point seeking to add other terms to the model. The LARS / LASSO algorithm is typically terminated when the residual error reaches a target noise floor, or when no additional model terms are able to enter the model. The sequence of basis function selection steps results in a set of models that are piecewise continuous in their $l_1$-norm. The target solution for an intermediate $l_1$-norm is obtained by interpolating along the solution path. In the event the solution terminates before reaching the target $l_1$-norm, the maximum $l_1$ value is instead accepted as the target solution. In addition, the LARS / LASSO solution framework is easily adapted to the requirements of the hyperspectral linear mixture model problem by only allowing model terms to enter the active set if they have coefficients with positive values. This restriction, combined with the above-described $l_1$-norm constrain, results in a mixture model solution with positive fill fractions whose sum is less than or equal to a target value $\gamma = 1.0$.

3. TARGET IDENTIFICATION ALGORITHM

The core concepts of the GeoID algorithm are shown in Figure 4. Primary inputs include HSI chip data pre-corrected to reflectance and a signature library in matching reflectance units. The first processing step is to create a background pixel mask for estimating statistics and endmember basis functions. The cued target pixel’s data spectrum is used to construct a matched filter which is applied to the entire chip HSI data. Any chip pixel with a target self-matched filter SCR value less than a threshold is added to the background mask. This step is critical in order to avoid “contaminating” background descriptors with target-correlated signals. The background is characterized with first- and second-order statistics in combination with scene derived endmembers. The target is described as a linear combination of a single library signature plus the set of background endmembers. GeoID processing is applied to candidate target pixel vectors, background endmembers and library signatures that have been whitened according to

$$\tilde{x} = K^{-1/2} (x - m)$$

where $m$ and $K$ are respective scene-derived estimate for the mean spectrum and covariance matrix.
The GeoID multi-hypothesis test begins by fitting the target with a background-only null-hypothesis model where the resulting fitting error is given by

\[ \chi^2_{\text{null}} = \left\| \mathbf{y} - \mathbf{X}_{\text{bkg}} \mathbf{\beta}_{\text{bkg}} \right\|^2. \]

The matrix \( \mathbf{X}_{\text{bkg}} \) is comprised of background endmembers and \( \mathbf{\beta}_{\text{bkg}} \) is the vector of corresponding background abundances estimated by the LARS/LASSO solver. Next, each signature in the library is visited in turn and used to augment the background model whose fitting error is given by

\[ \chi^2_{\text{sig},i} = \left\| \mathbf{y} - \mathbf{X}_{\text{sig},i} \mathbf{\beta}_{\text{sig},i} \right\|^2. \]

The matrix \( \mathbf{X}_{\text{sig},i} \) is equal to \( \mathbf{X}_{\text{bkg}} \) with an additional column vector corresponding to the signature for the \( i \)th material in the library. A practical model ranking metric is computed as the ratio of the normalized null-hypothesis fitting error relative to the normalized fitting error from each signature’s model,

\[ \phi_i = \frac{\chi^2_{\text{null}} / N_{\text{null}}}{\chi^2_{\text{sig},i} / N_{\text{sig},i}}. \]

The terms \( N_{\text{null}} \) and \( N_{\text{sig},i} \) are the respective model degrees of freedom computed as the number of spectral bands minus the number of active model terms. The “significance ratio” metric is interpreted as the factor by which a given signature improves the least-squares fit relative to the null-hypothesis background model. A signature with a significance ratio on the order of unity is not a notable improvement compared to the background model. The top-ranked library signatures above a pre-determined threshold are accepted as potentially valid target models. Note that in the case where the LARS/LASSO models maintain identical background terms, and the \( i \)th library signature is in fact added to the model, then the above “significance ratio” is linearly related to the classic F-statistic used in testing the significance of adding a new term to a model.

Figure 5 shows an example of fitting a target observation with a null-hypothesis model, followed by an incorrect target model and the correct target signature. The goodness-of-fit produced by the incorrect target model is only marginally improved compared to the null-hypothesis background-only model.

Figure 5. Example multi-hypothesis target identification. Linear mixture model with LARS / LASSO constrained fit shows significantly improved fit for correct target model.
4. PERFORMANCE EVALUATION

4.1. Simulated Targets with AVIRIS Data

Publicly available reflectance imagery from NASA’s AVIRIS airborne hyperspectral sensor was used to demonstrate GeoID false-alarm mitigation performance through post-processing identification of detected target cues. The data was spectrally sub-selected from the original 220 bands to 150 bands in order to exclude atmospherically opaque regions. A false-color RGB image of the scene is shown in Figure 6.

Real data containing both ground-truthed targets and false alarms is not available for this type of analysis as we require large numbers of target observations. An intermediate solution was implemented by artificially adding sub-pixel material signatures to randomly selected pixels from the AVIRIS data. Two types of target pixels were simulated using library reflectance signatures for two commonly used green paints, shown in Figure 7-a as Paint-A and Paint-B. False alarms were simulated using 60 signatures for randomly selected manmade materials from the publicly-available ASTER v2.0 spectral library. These false-alarm signatures (shown in Figure 7-b) included materials such as concrete, asphalt, colored plastics, other paints, and other common construction materials. Simulated target and false-alarm observations were generated using a simple replacement model:

\[ y = (1 - f) \cdot x + f \cdot (t + n). \]

The spectrum \( x \) is a randomly selected AVIRIS pixel, \( t \) is the artificial signature added to the observation, \( f \) is the simulated target’s sub-pixel fill fraction. The spectrum \( n \) is a random noise instance with a standard deviation computed as the square-root of the scene’s median eigenvalue. This noise term is necessary to maintain similar noise statistics between the unmodified background test data and the modified artificial target data. The sub-pixel fill fraction \( f \) was set to a constant value of 0.10 for all results described in this report.
A total of four data sets were derived from the AVIRIS data to support this performance analysis:
1. **Background Data**: 100,000 randomly-selected pixels from the Moffet Field AVIRIS reflectance cube.
2. **Paint-A Data**: Copy of Background Data where the replacement model was used to simulated an observation of Paint-A at sub-pixel fraction $f = 10\%$.
3. **Paint-B Data**: Same as above, but using the signature for Paint-B.
4. **False Alarm Data**: Copy of Background Data where each pixel is contaminated with a material selected randomly from the false-alarm library at fill fraction $f = 10\%$.

The Background Data represents imagery that a sensor would observe in the absence of any target or false alarm. Scene clutter statistics are computed solely from the Background Data and then used with both the Target Data and False Alarm Data for target detection performance testing. This is representative of detection in the presence of rare small targets, where the targets themselves do not contribute significantly to global scene statistics. Performance testing with this data involved comparing both detection and identification results from the Target Data sets versus the False Alarm Data set.

### 4.2. Identification Performance

GeoID was applied to the above simulated target data using statistics and endmembers derived from the Background Data set. The spectral library used for identification testing contained signatures for Paint-A, Paint-B, ten additional paint and fabric signatures, the 60 manmade materials used for simulating the false alarm observations, plus all remaining spectra from the ASTER database that spanned the same spectral range as the AVIRIS data. This final test library consisted of a total of 1353 signatures.

Material identification was performed for each simulated target and false-alarm observation by studying the ranked set of GeoID-computed significance ratios generated for each material in the large library. All library materials yielding a significance ratio greater than 1.05 were declared as acceptable. This is analogous to accepting models with at least a 5% improved fit relative to the null-hypothesis background model. The remaining signatures (if any) were ordered by significance ratio and the top-ranked material was selected as the pixel’s GeoID-determined “label”.

An identification confusion matrix is shown in Table 1 where the GeoID results are compared to the modeled true signatures. The category named Random is a grouping of all false-alarm signatures. Paint-A was correctly identified as itself for 79.3% of the 100,000 simulated observations. Only 59.3% of the Paint-B targets were correctly identified. For the randomly-generated false-alarm targets, almost 80% were identified as a material other than Paint-A or Paint-B, while 17.9% were identified as nothing at all.
4.3. False Alarm Mitigation

The standard adaptive matched filter (MF) and the Adaptive Coherence Estimator (ACE) detection algorithms were used to test performance improvements through false-alarm mitigation using GeoID. Note that ACE is easily implemented as the matched filter with each scene pixel normalized to a unit vector. Detection performance is quantified in the form of a ROC curve where on-target detector values are compared to off-target detector values over a range of threshold values. The probability of detection ($P_D$) is computed as the fraction of on-target observations that are detected for a given threshold. The probability of false-alarm ($P_{FA}$) is computed as the fraction of non-target observations that pass the same threshold values. The ROC curve is mapped as the detector threshold is varied from a minimum value to a maximum.

Baseline ROC curves for Paint-A and Paint-B, for both MF and ACE, are indicated in Figure 8 by the red lines. The solid lines represent matched filter results, while the dashed lines represent ACE results. Note that MF and ACE perform similarly for Paint-A, while ACE greatly outperforms MF for Paint-B. Interestingly, Paint-B is darker than Paint-A (Figure 7) and thus it appears that the target pixel magnitude normalization performed by ACE improves performance by forcing the match to be solely based on color, rather than both color and magnitude as done in MF.

GeoID was next applied as a post-processing step to all detections results derived from the Paint-A, Paint-B data, and the False-Alarm data, while testing for identification against the same large 1353–member signature library described in the previous section. Detection post-processing with GeoID simply involved noting whether or not the top-ranked GeoID signature was the same as that used during the initial detection process. For example, while applying a Paint-A matched filter to both the Paint-A Data and the False Alarm Data, any detection result was rejected if it was not also identified by GeoID as Paint-A. The gold ROC curves in Figure 8 illustrate the effect of this additional processing. The maximum $P_D$ is reduced by 20% for Paint-A and 40% for Paint-B. While a reduction in overall $P_D$ is undesirable, the improved false-alarm rate far outweighs this penalty. The false-alarm rate for Paint-A is improved by almost two orders of magnitude, while the Paint-B false-alarm rate is only improved by a single order of magnitude. Application to real-world detection problems are not expected to be as dramatic as this simulation suggests as real target observations may not be as well matched to their library signature counterparts.

5. CONCLUSIONS

GeoID is a new sub-pixel target identification algorithm implementing a physically-constrained linear mixture model with the LARS/LASSO model-building framework. It is proposed as a means for false-alarm mitigation whereby candidate detection cues are tested for consistency with top-ranked identification results. Multi-hypothesis identification testing allows for efficient testing against large signature libraries and comparison with a background-only null-hypothesis model. Performance testing with AVIRIS data plus simulated target and false-alarm signatures indicates potential for significantly reduced false-alarm rates at the expense of modest reduction in positive detection rates.
Figure 8. ROC curve performance results illustrating reduced false-alarm rates when GeoID is used as a detection post-processing validation step.
6. REFERENCES


